Int. J. Heat Mass Transfer. Vol. 15, pp. 1185-1202. Pergamon Press 1972. Printed in Great Britain

PIONEER PAPER

Preface to the Paper by N. P. KASTERIN
GENERALIZATION OF BASIC EQUATIONS OF AERODYNAMICS AND ELECTRODYNAMICS

A. S. PREDVODITELEV
Moscow State University, Moscow, USSR

and

V. A. BUBNOV
Heat and Mass Transfer Institute, BSSR Academy of Sciences, Minsk, USSR

1. Nikolai Petrovich Kasterin was born on the 14th of December 1869 in the Kaluga Guberniya. He received his primary education at the Trestin people school and the Zhizdrin progymnasium. In 1888 he entered Moscow University from which he graduated in 1892. He was a pupil of the eminent Russian physicist A. G. Stoletov. From 1896 till 1899 N. P. Kasterin was abroad and worked at the Berlin Physical Institute under Professor E. Warburg.

In 1900 N. P. Kasterin came back to Moscow and began to teach at Moscow University. In 1905 the Doctor of Physics Degree was conferred on him for the dissertation "On Wave Propagation in a Nonhomogeneous Medium". Thereupon N. P. Kasterin moved to Odessa to work at the Novorossiysk University as a Professor of Physics.

In 1920 he returned to Moscow and to his last days was connected with Moscow University at the same time collaborating with several research institutes.

N. P. Kasterin died on the 10th of December 1947.

2. Little is known of N. P. Kasterin's scientific activity and he is famous only for his works on acoustics.

Kasterin discovered dispersion of acoustic waves propagating in nonhomogeneous media. In 1898 his preliminary investigation on dispersion of acoustic waves was reported by Kamerlingh-Onnes at the Academy of Sciences in Amsterdam and published there. In 1900 the second communication was published in Leiden in G. A. Lorentz dedication volume. The investigation results were published in full in Moscow. in 1903.

N. P. Kasterin set a task of investigating wave propagation in a nonhomogeneous medium in a most general form. From the concept of optical media as consisting of a continuum (ether) and molecules (resonators) distributed in it, Kasterin suggested an analogous model of a nonhomogeneous medium for acoustic waves consisting of a continuum (air) and bodies (resonators) placed there in a definite order.

A similar model was used by Rayleigh who, however, treated only the limiting case when the wave length was infinitely great in comparison to the period of the structure of a nonuniform medium. According to Kasterin's theory, Rayleigh's result is recognized as the simplest particular case.

The basic conclusion to which Kasterin came lies in the following: Laws of wave propagation in a nonhomogeneous medium of any mode may be formulated similarly to the laws of wave propagation in a homogeneous medium if two characteristics are introduced for the nonhomogeneous medium, namely, η—refraction coefficient and ρ—medium density (fictitious). Both characteristics depend not only on the medium structure but also on the wave length λ of propagating waves.

Kasterin did not confine himself to theoretical research. He undertook an extensive experimental investigation aimed at confirmation of the theory suggested by him. Experimental results demonstrated an excellent coincidence between theory and experiment. Measurements showed distinct regions of anomalous dispersion predicted by the theory.

It should be noted that a phenomenon of anomalous dispersion in optics and its relationship with absorption were established in 1895, i.e. shortly before Kasterin's works, by Pfluger who had measured a refraction coefficient of light in a prism of solid fuchsin for waves of different length including an absorption band of fuchsin.

Thus, Kasterin experimentally established a complete analogy between the disperse phenomena in optics and acoustics.

This result was emphasized by P. N. Lebedev who contributed a great deal towards establishing a common character of wave processes of different nature. In the article "Advances in Acoustics in the Last Decade" he mentioned that for the last years to interference and diffraction phenomena of sound, testifying to common kinematic properties of wave processes in acoustics and optics, there was added one more phenomenon of sound influence on a refracting wall which is analogous to that of light pressure on mirror surfaces. He wrote: "But a greater achievement in this direction belongs to N. P. Kasterin who was able to observe the analogy of dispersion phenomena of acoustic and light waves."

Below is given a list of works by N. P. Kasterin composed by Kasterin and kindly placed at our disposal by his daughter T. N. Kasterina. Many of the papers from the list were reported to the Lebedev Physical Society and are unpublished. Their fate is as yet unknown.

3. The work by N. P. Kasterin "Generalization of Basic Equations of Aerodynamics and Electrodynamics" was originally reported in complete form at the USSR Academy of Sciences on the 9th of December 1936. However, as is known from Kasterin's letters to the famous Russian professor of mechanics N. E. Zhukovsky, the basic results dealing with electrodynamics were obtained by 1917.

This paper is written in a very condensed form; almost all the intermediate calculations are omitted in it. Concerning the method the following is only said: "When trying to find the second approximations, we should bear in mind not only that it deals with velocities equal and even higher than those of sound and, accordingly light but also the fact that after Euler's and Maxwell's time there were established experimental facts which change radically the very interpretation of those problems. I mean, the fact of discontinuity of the structures both of gas and electric field. In Euler's and Maxwell's derivation, on the contrary, continuity was postulated in these cases. Discontinuity of gas structure is established primarily by the kinetic gas theory and its agreement with facts, discontinuity of the electric field by the existence of an elementary electric charge (electron) ..."

"For the basic method when solving the problems set forth I make use of the Lagrange equations of dynamics extended to physical systems by Helmholtz and given in his last works on the principle of least action ..."

From equations (3) and (4) of the work under consideration it is seen that for vortex-free flow this methodology for the second approximation equations leads to a change of the sign before convective terms in the Euler equations. Details of the methodology Kasterin never published. We shall make an attempt to restore his methodology by deriving the second approximation equations in the Cartesian coordinate system.

Let us introduce finite-difference representations. For this purpose let an elementary cell of the network be removed (Fig. 1) and let the radius vector $\boldsymbol{r}_0$ determine the position of the centre of gravity of the liquid volume. As a result of motion the elementary volume may be deformed within certain limits but it should remain discrete, i.e. it cannot be tightened into a point. Next, let the hydrodynamic velocity vector $\boldsymbol{V}$ be constant within the elementary

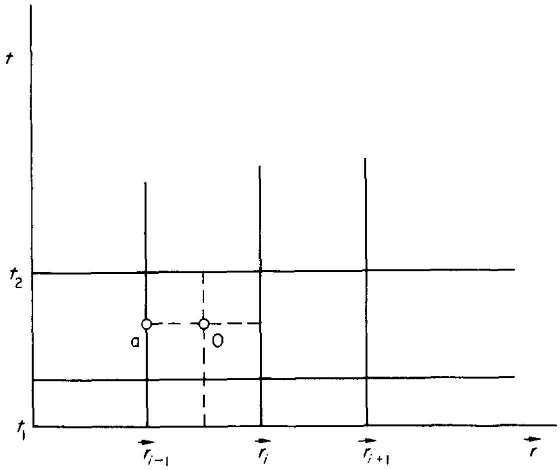

FIG. 1. Elementary network cell.

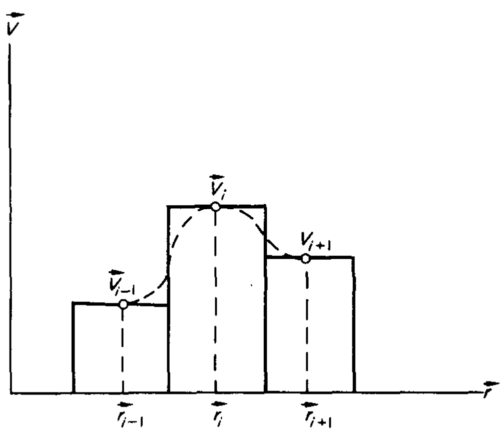

FIG. 2. Discontinuous velocity change.

volume and change stepwise at the boundary of the cell (Fig. 2). Then it is evident that within the cell boundaries we shall get

$$\frac{\partial \boldsymbol{V}}{\partial x_i} = 0\,; \frac{\partial \boldsymbol{V}}{\partial y_i} = 0\,; \frac{\partial \boldsymbol{V}}{\partial z_i} = 0\,. \qquad (1)$$

If we introduce coordinates of the centre of gravity of this volume $\boldsymbol{r}_{0i}$, then it will be clear that

$$\sim \frac{\partial \boldsymbol{V}}{\partial x_{0i}} \neq 0\,; \frac{\partial \boldsymbol{V}}{\partial y_{0i}} \neq 0\,; \frac{\partial \boldsymbol{V}}{\partial z_{0i}} \neq 0\,.$$

Applying finite-difference formulae, it can be written

$$\frac{\partial \boldsymbol{V}}{\partial x_{0i}} = \frac{\boldsymbol{V}_{i+1} - \boldsymbol{V}_i}{\Delta x_{0i}}\,; \frac{\partial \boldsymbol{V}}{\partial y_{0i}} = \frac{\boldsymbol{V}_{i+1} - \boldsymbol{V}_i}{\Delta y_{0i}}\,; \frac{\boldsymbol{V}_i}{\partial z_{0i}}$$

$$= \frac{\boldsymbol{V}_{i+1} - \boldsymbol{V}_i}{\Delta z_{0i}}\,. \qquad (2)$$

Here are introduced apparent notations

$$\left.\begin{aligned} \Delta x_{0i} &= x_{0i} - x_{0i-1} = \beta(x_{0i-1} - x_{i-1}) \\ \Delta y_{0i} &= y_{0i} - x_{0i-1} = \beta(y_{0i-1} - y_{i-1}) \\ \Delta z_{0i} &= z_{0i} - z_{0i-1} = \beta(z_{0i-1} - z_{i-1}) \end{aligned}\right\} (3)$$

wherein β indicates dimensions of the network cell.

Thus, treating the vector $\boldsymbol{V}$ as a function of the radius vector $\boldsymbol{r}_{0i}$, this function may be considered smooth (the dotted line in Fig. 2). For this case not the behaviour of this curve at points $\boldsymbol{r}_i$ is of importance but the fact that it approaches the points $\boldsymbol{r}_{0i}$ with the zeroth derivative so, that condition (1) is fulfilled. It is easy to note that condition (1) shifts the instantaneous centre of rotation from point "0" to points "a".

Considering the velocity vector $\boldsymbol{V}_i$ as the function of $\boldsymbol{r}_{0i}$, we may write an expansion series

$$\boldsymbol{V}_i = \boldsymbol{V}_{i-1} + \Delta x_{0i}\frac{\partial \boldsymbol{V}_{i-1}}{\partial x_{0i-1}} + \Delta y_{0i}\frac{\partial \boldsymbol{V}_{i-1}}{\partial y_{0i-1}} + \Delta z_{0i}\frac{\partial \boldsymbol{V}_{i-1}}{\partial z_{0i-1}}\,. \qquad (4)$$

Designating the elementary volume mass by m, let us determine its kinetic energy

$$T = \frac{mV_i^2}{2} = m\left[\frac{V_{i-1}}{2} + \frac{\Delta x_{0i}}{2}\frac{\partial V_{i-1}^2}{\partial y_{0i-1}} + \frac{\Delta y_{0i}}{2}\frac{\partial V_{i-1}^2}{\partial y_{0i-1}} + \frac{\Delta z_{0i}}{2}\frac{\partial V_{i-1}^2}{\partial z_{0i-1}}\right], \quad (5)$$

herein the velocity vector squared is determined through projections u, i, w as

$$V_{i-1}^2 = u_{i-1}^2 + v_{i-1}^2 + w_{i-1}^2. \quad (6)$$

The potential energy of an ideal liquid is controlled only by elastic forces of pressure

$$\Pi = m\int_{p_0}^{p} \frac{dp}{\rho(p)}. \quad (7)$$

On introducing the generalized coordinates $x_{0i-1}, y_{0i-1}, z_{0i-1}$ and the generalized velocities

$$\frac{dx_{0i-1}}{dt} = u_{i-1}; \frac{dy_{0i-1}}{dt} = v_{i-1}; \frac{dz_{0i-1}}{dt} = w_{i-1},$$

the system of the Helmholtz equations for the elementary volume will take the form

$$\left.\begin{aligned} \frac{\partial H}{\partial x_{0i-1}} - \frac{d}{dt}\frac{\partial H}{\partial u_{i-1}} &= 0 \\ \frac{\partial H}{\partial y_{0i-1}} = \frac{d}{dt}\frac{\partial H}{\partial v_{i-1}} &= 0 \\ \frac{\partial H}{\partial z_{0i-1}} - \frac{d}{dt}\frac{\partial H}{\partial w_{i-1}} &= 0. \end{aligned}\right\} \quad (8)$$

Suppose, that rot $V_{i-1} = 0$, then we shall get

$$\left.\begin{aligned} -\frac{d}{dt}\frac{\partial H}{\partial u_{i-1}} &= m\left[\frac{\partial u_{i-1}}{\partial t} + \frac{1}{2}\frac{\partial}{\partial x_{0i-1}}(u_{i-1}^2 + v_{i-1}^2 + w_{i-1}^2)\right] \\ \frac{\partial H}{\partial x_{0i-1}} &= -\frac{m\beta}{2}\frac{\partial V_{i-1}^2}{\partial x_{0i-1}} + \frac{m}{\rho}\frac{\partial p}{\partial x_{0i-1}}. \end{aligned}\right\} \quad (9)$$

Substituting formulae (9) into the first equation (8), we get

$$\frac{\partial u_{i-1}}{\partial t} + (1-\beta)\frac{\partial}{\partial x_{0i-1}}\left(\frac{u_{i-1}^2 + v_{i-1}^2 + w_{i-1}^2}{2}\right) = -\frac{1}{\rho}\frac{\partial p}{\partial x_{0i-1}}. \quad (10)$$

Building analogous equations for other axes, assuming $\beta = 2$ and dropping out subscripts $(0i-1)$, we get the following equation in vectorial form

$$\frac{\partial V}{\partial t} - \mathrm{grad}\,\frac{V^2}{2} + \frac{1}{\rho}\mathrm{grad}\, p = 0. \quad (11)$$

Equation (11) corresponds to the set of equations (4) in the work by N. P. Kasterin.

Using modern concepts on discontinuous functions in a most general form, adhering to Kasterin's methodology, the following equations for viscous liquid flow [2] may be obtained

$$\rho\frac{\partial V}{\partial t} + \rho\,[(1-\beta)(V\nabla)V - \beta V \,\mathrm{div}\, V] = -\mathrm{grad}\, p + \frac{\partial}{\partial x}\left[\mu\left(\frac{\partial V}{\partial x} + \mathrm{grad}\, u\right)\right] + \frac{\partial}{\partial y}\left[\mu\left(\frac{\partial V}{\partial y} + \mathrm{grad}\, v\right)\right] + \frac{\partial}{\partial z}\left[\mu\left(\frac{\partial V}{\partial z} + \mathrm{grad}\, w\right)\right] + \mathrm{grad}\,(\lambda\,\mathrm{div}\, V). \quad (12)$$

Here by μ and λ, respectively, the coefficients of bulk and shear viscosity are designated.

Since within the limits of the cell the discontinuous function mentioned may be smoothed with different accuracy, then the very parameter β which has appeared in equation (12) characterizes the uniqueness of smoothing.

Equations (12) were obtained by A. S. Predvoditelev from molecular-kinetic representations in 1948 [3].

4. We shall not analyze the electrodynamic part of the work by N. P. Kasterin but note that a reader abroad can become interested in this forgotten paper in connection with some views expressed by the famous physicist of our century P. A. M. Dirac in 1963 [1].

P. A. M. Dirac's opinion is so important for services of N. P. Kasterin to be recognized that we shall quote some of them from the Russian translation of his article "Evolution of the Physical Picture of Nature":

"I would like to mention another possible physical picture referring to the question why all electrical charges observed in nature must be multiple to the elementary charge e (e is an electron charge, note of Editor). Why does a charge nowhere in nature propagate continuously? I suppose a physical picture that represents the further development of the former ideas concerning the Faraday force lines. The Faraday force lines distinctly represent a picture of electric fields. If in some region of space there is an electric field, then according to Faraday, it may be represented as a system of lines, the direction of which at every point coincides with that of the electric field. Density of field lines shows an electric field force: the closer the lines approach each other, the stronger is the field and vice versa. The Faraday field lines give us an excellent picture of an electric field from the point of view of the classic theory."

"If we imagine that these discrete Faraday field lines represent something fundamental in physics and serve as the basis for an electromagnetic field picture, then it will be clear why values of charges are always multiples of e. Explanation lies in the following: Since the end of a field line corresponds to every particle then a number of field lines should be integral. Thus, we arrive at the physical picture which is quite satisfactory from the qualitative point of view."

"Sometimes breaking of a field line may occur. When it takes place, two ends appear which should be charges. Such a process can be imagined as a picture of formation of an electron $(-e)$ – positron $(+e)$ pair. This picture is quite reasonable and if one succeeded in its developing, it would give a theory in which the charge would enter as a fundamental value. I could not manage to find any reasonable system of equations for motion of these force lines and simply propose this idea as a possible physical picture."

N. P. Kasterin took this very idea as a basis for his equations of an electromagnetic field as far back as 1917, and as a result of embodiment of this idea he wrote in 1937:

"Comparing the solutions obtained for electron (positron) and proton, we may predict that if we manage in an experiment, with the aid of an external magnetic field, to increase the velocity of positron rotation by $1838/2 = 919$ times, it will be converted into a proton, and an electron in a similar way must be converted into an anti-proton, which as yet has not been obtainable by experimental physicists.

Analogous views were expressed by Prof. N. E. Zhukovsky in the report "Obsolete Mechanics in Modern Physics" read on the 3rd of March 1918 in the Moscow Mathematical Society. We cite some of them.

"Professor N. P. Kasterin in his report to the Petrograd Academy of Sciences dealing with an analysis of Bukherer's experiments with flight of β particles emitting from radium shows disagreement between experiments and Einstein's formula. To develop the mechanics of ether, in my opinion, it is necessary to perceive mechanical construction of the Maxwell equations not avoiding the classical procedure which Thomson and Helmholtz followed. Any physical system, the total energy of which is expressed by parameters and their derivatives, may be treated with the help of the Lagrange equations and Hamilton principle, though details of the system construction may be unknown. Some cyclic processes, which cannot be observed, may proceed in it that obscure a phenomenon on the face of them."

"Derivation of the Maxwell equations in the the classical form mentioned is performed by Professor N. P. Kasterin. With the author's permission I shall finish my speech by emphasizing the idea of this derivation. The first group of Maxwell equations appears to express constancy of the Faraday tubes, it is quite analogous to the law of conservation of vortices in hydrodynamics of incompressible liquids, electric force lines are recognized here as vortex lines; an elementary charge is vortex stress and ether displacement plays the role of velocity. If we consider the Faraday tube system and compute its total energy by the known equation for the energy of an electromagnetic field, then the Hamilton principle will give the second group of Maxwell equations expressing a time derivative of magnetic forces with some generalization, that leads to general equations for the case of perpendicular displacement towards electric forces. Does an analogy of the Faraday tubes with vortices of incompressible liquid give the way for construction of ether mechanics or not? Has mechanics really lost its role in the new physics?"

REFERENCES

1. P. A. M. DIRAC, Evolution of physical picture of nature, Collected papers, *Elementary Particles*, vyp. 3. Izd. Nauka (1965).
2. V. A. BUBNOV, One comment to special solutions of hydrodynamic equations, *Inzh.-Fiz. Zh.* No. 1 (1970).
3. A. S. PREDVODITELEV, One molecular-kinetic foundation of hydrodynamics equations, *Izv. AN SSSR, Otd. Tekh. Nauk.* No. 4 (1948).

LIST OF SCIENTIFIC WORKS BY N. P. KASTERIN

1. On temperature change of liquids adhesion, *Zhur. R. F. Kh. O*, (1892) (the work is awarded by the Physical Society).
2. A change of surface tension of ethyl ether with temperature, *Zhur. R. F. Kh. O.* (1893).
3. Determination of capillary constant by droplet sizes, *Zhur. R. F. Kh. O.* (1895).
4. Sound dispersion, *Zhur. R. F. Kh. O.* (1897) (the work is awarded by the Physical-Mathematical Faculty of the Moscow University).
5. Polar region of natural science (speech), Newspaper "*Russkie Vedomosti*", No. 266 (26 November, 1898).
6. On acoustic waves dispersion in a non-homogeneous medium (Preliminary Communication), Dimakov Printer in Petersburg (1898).
7. Über die Dispersion der akustischen Wellen in einem nicht homogenen Medium (Reported in the Academy of Sciences in Amsterdam, 1898).
8. Über die Ausbreitung der Wellen in einem nicht homogenen Medium von lamillar Structur (Reported in H. A. Lorentz's dedication volume((1900).
9. On wave propagation in non-homogeneous medium, *Izd. Mosk. Univ.* (1903).
10. Matter and Electricity (1906) (speech).
11. Sur une contradiction on essentielle entre la theorie de relativité d'Einstein et l'experience Bulletin de l'Acad. der Scien. de Ruth'e (1918).
12. On incompetence of Einstein's relativity principle, Uchenye Zap. of Novorossiysk University, Odessa (1919).
13. Mechanical part of physics (Course of Lectures), Odessa (1919).
14. On gyromagnetic ratio (1921).
15. To the theory of anchor ice formation (reported at the Physical Society) (1924).
16. On a considerable theoretical error in works by Einstein, de Haas and Barnett, discovering the existence of molecular currents in a magnet (reported in the Lebedev Physical Society, 1923), Lomonosov Moscow State University Archives.
17. Newton Mechanics and Maxwell Electrodynamics (speech at the Lebedev Physical Society, 1925).
18. Pneumatic absolute electrometer (patent 1926).
19. On Sir T. T. Thomson's model of a light quantum, Philosophical Magazine (December 1926).
20. Derivation of basic relationships of the quantum theory from Maxwell equations, *Zhur. R. F. Kh. O.* **VIII** (ZA), p. 285–288 (1926).
21. Peculiarities of a light quantum in T. T. Thomson's investigations (reported at V Congress of Physics in Moscow, 1926).
22. The light quantum theory and Maxwell equations (reported in the Lebedev Physical Society, 1927).
23. On vortex motion occurrence (reported in the Lebedev Physical Society, 1929).
24. Electron waves and the T. T. Thomson theory (reported in the Lebedev Physical Society, 1930).
25. Generalization of methematical formulation of the light aberration law and Doppler principle and its effect on the theory of Michelson and Daiton-Millers experiments, Report of the USSR Academy of Sciences, pp. 226–235 (1932) (presented by Acad. A. S. A. Chaplygin).
26. Vibration method for strength tourills testing (the Mendeleev Institute Archives, 1931).
27. Mathematical theory of thermal resistance of ceramic tubes (1933).
28. New measuring method of thermal conductivity of poor conductors (1934).
29. Generalization of basic equations of aerodynamics and electrodynamics, *Izd. AN SSR* (1937).
30. On unusual peculiarity of heavy particles (the Lomonosov Moscow State University Archives, 1937).
31. Elimination of the F. Klain aerodynamical paradox, *Vestnik Mosk. Gos. Univ.* No. 10 (1949).
32. Experimental preparation of an air vortex column, written with A. K. TIMIRYAZEV and T. M. SVIRIDOV, *Vestnik Mosk. Gos. Univ.* No. 10 (1949).
33. Vortex hysteresis, written with A. K. TIMIRYAZEV and T. M. SVIRIDOV, *Vestnik Mosk. Gos. Univ.* No. 10 (1949).